

The Effectiveness of Delta Hedge

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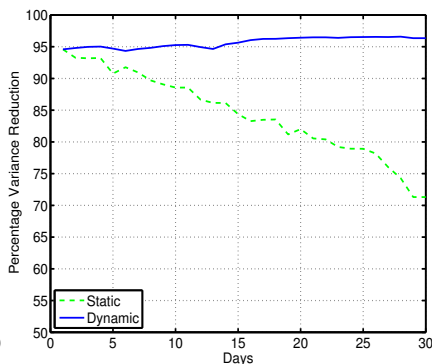
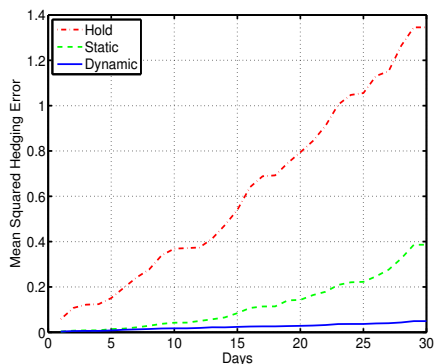
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How (in)effective is the Black-Merton-Scholes model?

- If you sell an option, the BMS model says that you can completely remove the risk of the call by continuously rebalancing your shareholdings to neutralize the delta of the option.
- We perform the following experiment using historical S&P 500 options data from Jan 1996 - Jan 2015:
 - Sell a one-month at-the-money call option
 - Record the P&L from three strategies:
 - 1 *Sell and Hold*: Just hold the option short.
 - 2 *Static Delta Hedge*: Perform a delta hedge initially, but do no re-hedging.
 - 3 *Daily Delta Hedge*: Restore the delta to zero at the end of each day.
- We record the P&L each month, then calculate the variance of P&L for each of the 3 strategies over the 16 year period.
- By what fraction do you think the variance of P&L is reduced as we switch from sell-and-hold to daily delta hedge?

[A] No reduction, [B] 0 to 1/3 , [C] 1/3 to 2/3 , [D] 2/3 to 100%

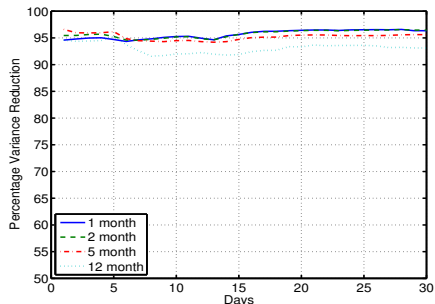
Variance reduction for written 30-day at-the-money call



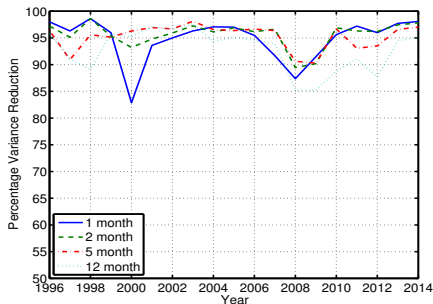
- Unhedged, the standard deviation of P&L can exceed the option price.
- Un-rebalanced, the static delta hedge deteriorates over time, but still removes 70% of the variance over one month.
- Rebalanced daily, 95% of the variance can be removed on average over our sample period.
- The answer is “D.”

Hedging ATM calls at different maturities and time periods

Different maturities



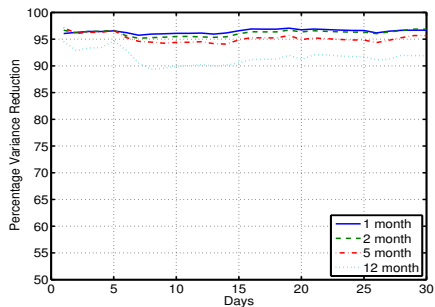
Different years



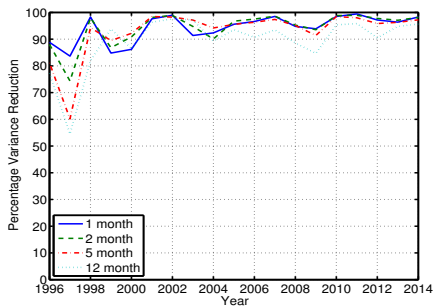
- Dynamic delta hedge works equally well on both short and long-term call options.
- To gauge the stability of this result over time, we calculate the variance reduction for one month P&L for different calendar years
- The reduction in variance deteriorates somewhat in times of crisis.

Hedging ATM puts at different maturities and time periods

Different maturities



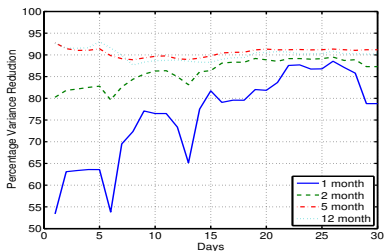
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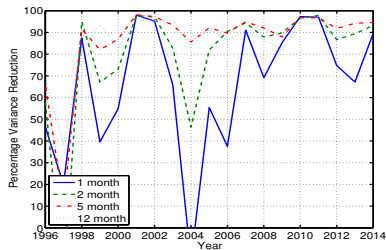
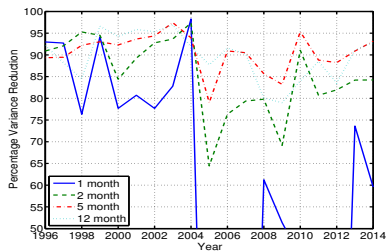
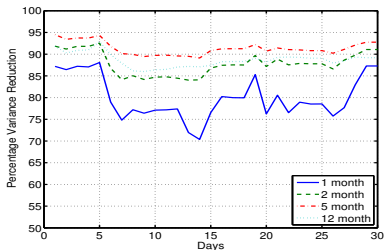
- Dynamic delta hedge works equally well on both short and long-term puts.
- The reduction in put option variance works well in the recent financial crisis, but some deterioration in early 1997.

Hedging 95% OTM options

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P



- Improvements are smaller if the options are too far out of money and never get close to the money, and the variance is small even without hedging.